Revision 1

Year 11 Examination

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section One:
Calculator-free

Student Number: In figures

In words ______________________________________

Teacher name ______________________________________

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **35% (52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

Two vectors are given by $\mathbf{a} = 9\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$. Determine

(a) a vector parallel to $\mathbf{a} - \mathbf{b}$ of magnitude 25. (3 marks)

(b) $\mathbf{a}$ in terms of $\mathbf{d}$ and $\mathbf{e}$, where $\mathbf{d} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{e} = 5\mathbf{i} - 2\mathbf{j}$. (4 marks)

Question 2 **(7 marks)**

Three vectors are given by $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$.

Determine

(a) a unit vector $\mathbf{d}$, parallel to $\mathbf{a} + 2\mathbf{b}$. (3 marks)

(b) the value(s) of k so that the magnitude of the vector $\mathbf{a} + k\mathbf{b}$ is 4. (4 marks)

Question 3 **(9 marks)**

Consider the matrices $A = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & -5 \end{bmatrix}$.

(a) It is possible to form the product of all four matrices. State the dimensions of the resulting product. (2 marks)

(b) Determine the matrix $\frac{1}{2}DC$. (2 marks)

(c) Determine the inverse of matrix A. (2 marks)

(d) Clearly show use of matrix algebra to solve the system of equations $2x - 3y + 3 = 0$ and $4y = 2x + 2$. (3 marks)

Question 4 **(7 marks)**

(a) Matrix A represents a rotation of 180º about the origin. Determine

(i) matrix A. (1 mark)

(ii) the exact coordinates of the point (-2, 3) after transformation by matrix A. (1 mark)

(iii) the determinant of matrix A. (1 mark)

(b) Matrix $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Describe the transformation represented by B and calculate its determinant. (2 marks)

(c) Use an example to show that two non-singular square matrices C and D exist such that the determinant of their sum is equal to the sum of their determinants. (2 marks)

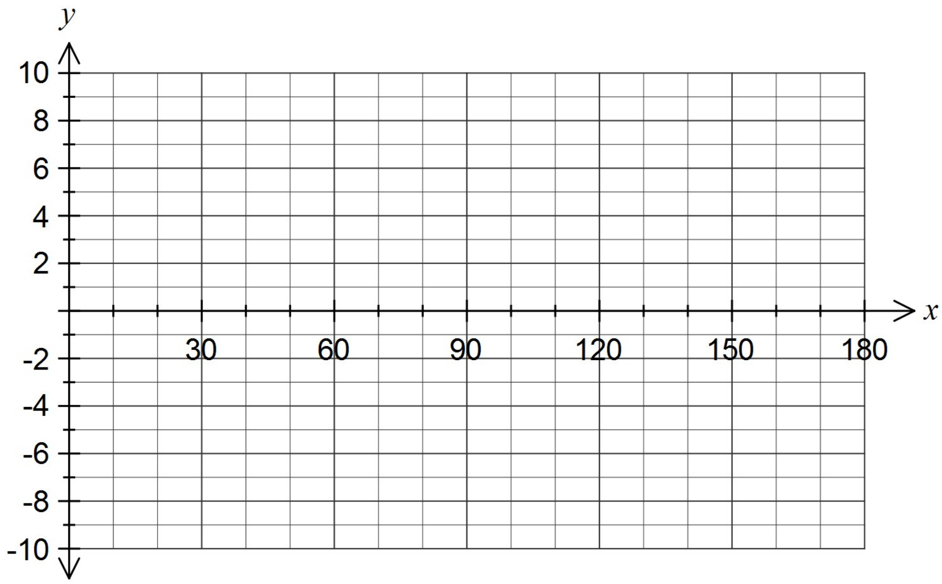
Question 5 **(7 marks)**

(a) Solve the equation $\tan\left(\frac{x+25°}{2}\right) = \sqrt{3}$ for $0° \leq x \leq 540°$. (3 marks)

(b) Prove that $(1 - \cos x)(1 + \sec x) = \sin x \tan x$. (4 marks)

Question 6 **(7 marks)**

(a) Sketch the graph of $y = 2\,\mathrm{cosec}(x + 90)$ for $0° \le x \le 180°$. (3 marks)



(b) Prove the identity $\cot A + \tan A = \sec A\,\mathrm{cosec}\,A$. (4 marks)

Question 7 **(8 marks)**

(a) Prove that the sum of any three consecutive terms of an arithmetic sequence with first term a and common difference d is always a multiple of three, for $a, d \in \mathbb{N}$. (3 marks)

(b) Use mathematical induction to prove that $7^{2n-1} + 5$ is always divisible by 12, for $n \in \mathbb{N}$.
(5 marks)